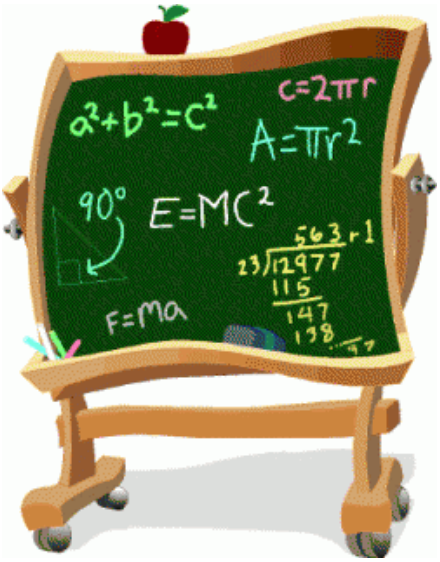
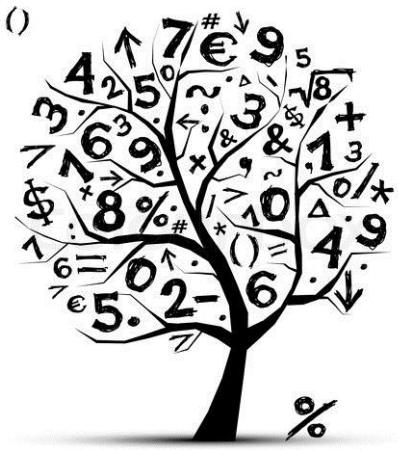


King Edward VI

Numeracy Handbook



Anna Dowen in collaboration with Maths Department and Numeracy Working Party

King Edward VI School Lichfield

Numeracy Handbook

This handbook has been designed to give guidance and help to staff, students and parents. It covers the methods used by the Maths department, and throughout the school. It is hoped that by having a consistent approach to these fundamental skills it will be easier for students to transfer these skills across the curriculum, and make greater progress.

Whenever possible mental methods should be encouraged, and the ability to use written methods checked. Students should all be encouraged to have a calculator with them at all times, and follow the correct methods when using them. Students should always be encouraged to 'estimate' answers before working them out to get an idea of the expected magnitude of their answers.

In some cases different methods will be given which will be used depending on the ability of the students. The methods will always start with the simplest, with more advanced students aiming to master the more sophisticated methods.

For any further assistance or clarification please discuss with a member of the Maths department, or Numeracy Working Group.

Table of Contents:

NUMBER

Addition

Mental: Three common strategies below for 63 + 49:

- Add tens and then units
  60 + 40 + 3 + 9 = 100 + 112 = 112
- Split number adding into tens and units
  63 + 40+ 9 = 103 + 9 = 112
- Round number adding and subtract
  63 + 50 – 1 = 113 – 1 = 112

Written: Column addition

- Make sure numbers always put into the correct column
- Always start addition at the right hand side

Example 1: 63 + 49

$$\begin{array}{r} 63 \\ +\ 49 \\ \hline 113 \\ {\scriptstyle 1} \end{array}$$

Example 2: 43.8 + 4 + 23.76

$$\begin{array}{r} 43.80 \\ 4.00 \\ +\ 23.76 \\ \hline 71.56 \end{array}$$

Note: Remainders can be below answer line, or below lowest value

Subtraction

Mental methods: Two strategies used below for 83-46

- Count on from 46 to 83

46 + 30 + 7 = 83 so 83 – 46 = 37

- Split number being taken into tens and units

83 – 40 – 6 = 43 – 6 = **37**

Written method: All students should be competent using the column subtraction.

Ensure numbers lined up correctly. Start on the left hand side, borrow if subtraction cannot be done.

Example 1: 4590 – 386

$$\begin{array}{r} {}^{8}4{}^{1}590 \\ -\ 386 \\ \hline 4204 \end{array}$$

Example 2: 14597 – 692

$$\begin{array}{r} 1{}^{3}{}^{1}4597 \\ -\ 692 \\ \hline 13905 \end{array}$$

Example 3: 5000 - 327

$$\begin{array}{r} {}^{4}5{}^{9}0{}^{9}0{}^{10} \\ -\ 327 \\ \hline 4673 \end{array}$$

If subtracting with DECIMALS ensure the decimal points are lined up.

Eg 20.90 – 8.36

$$\begin{array}{r} {}^{1}{}^{1}{}^{8}{}^{1} \\ 20.90 \\ -\ 8.36 \\ \hline 12.54 \end{array}$$

Multiplication

All times tables 1-10 should be known.

Mental methods of multiplying:

by 2 – double

by 4 – double and double again

by 8 – double three times

by 10 – move all digits one place to the left.

Written methods:

Grid/table method (all foundation, lower high students)

Eg1 123 x 5

x	100	20	3
5	500	100	15

500
+ 100
+ 15
615

Eg2 23 x 15

×	20	3
10	200	30
5	100	15

$200 + 30 + 100 + 15 = 345$

Traditional Column method (higher ability students)

Eg1 53 x 24

53
x 24
212
1060
1272

Eg2 5127 x 4265

5127
x 4265
25635
307620
1025400
20508000
21866655

Multiplication involving decimals: Ignore the decimal points until after the multiplication

Eg 5.3 x 0.24 (3 digits after decimal point)

Work out 53 x 24 by method above= 1272

Replace decimal point ensuring 3 digits after decimal point

5.3 x 0.24 = 1.272

Division

Mental Methods to divide:

By 10 or 100 move digits 1, 2 places to the right

By single digits – work backwards from tables

eg to find 48 ÷ 6 think 6 x ? = 48 answer: 8

Written method – 'Bus Stop'

Eg1 192 ÷ 8

$$8\overline{)19^{3}2}\quad 24$$

Decimals calculate in exactly same way. Ensure decimal points line up.

Eg 4.74 ÷ 3

$$3\overline{)4.{}^{1}7^{2}4}\quad 1.58$$

Remainders: If number does not divide exactly add additional zeros and continue dividing

Eg 2.2 ÷ 8

$$8\overline{)2.{}^{2}2^{6}0^{4}0}\quad 0.275$$

Percentages

Finding% of….

Mental methods of finding %s

10% - divide by 10

5% - halve 10%

1% - divide by 10 and 10 again, (or 100)

50% - halve

25% - halve and halve again (or divide by 4)

Other values – add multiples of these

Calculator methods

- To find a% divide by 100 and multiply by a
  Eg 24% of £275 is 275 ÷ 100 x 24 = £66
- Multiply by decimal equivalent of the percentage
  Eg 24% of £275 is 275 x 0.24 = £66

Increasing/Decreasing a value by a given %

- Find the amount of increase or decrease and add/subtract from original
  Eg increase £275 by 24%
  25% is £66
  Increased value is 275 + 66 = £341
- Multiply by decimal multiplier
  Eg increase £275 by 24% we need 124% of original
  £275 x 1.24 = £341
  Eg decrease £275 by 24% we need 76% of original
  £275 x 0.76 = £209

Finding percentage change

% change = $\frac{new-original}{original\ amount}$ x 100%

Eg I buy a car for £4800 and sell it for £3600. What was the % loss?

% loss = $\frac{1200}{4800} \times 100\%$ = 25% loss

One value as a percentage of another

To find A as a percentage of B work out:

$$\frac{A}{B} \times 100\%$$

Rounding

Numbers can be rounded to give an approximation. Numbers must never be shortened without considering rounding.

To round:

1. Identify the place value to which we want to round (eg rounding to nearest 10, 2 decimal places, 3 significant figures)
2. Look at the digit to the right:
   - If less than 5 round down
   - If 5 or more round up
3. Ensure number correct size, add zeros as necessary

Examples:

1. Round 4562 to nearest 10

4562 number to right is less than 5 so round down - 4560

2. Round 0.0567 to 2 decimal places

0.0567 number to right is 5 or more so round up – 0.06

(NOTE: additional zeros not needed in this case)

3. Round 57852 to 3 significant figures

57852 number to right is 5 or more so round up – 58000

Standard Form (or Standard Index Form)

Very large or small numbers may be written more simply in Standard form:

A number expressed in standard form must be written as:

$a \times 10^{b}$ where a is 1-10 and b is any number

Examples:

Positive Power = Large Number

$$4.3 \times 10^{6} = 4\,300\,000$$

Negative Power = Small Number

$$2.1 \times 10^{-3} = 0.021$$

Note: Calculators often give answers in standard form.

SHAPE AND SPACE

Units of Measure

As much as possible students are to always use METRIC units of measurements. They should be aware of some conversions to common Imperial units.

The following are the conversions all students should be familiar with:

Metric

1 km = 1000m 1 litre = 1000 ml 1 tonne = 1000 kg

1 m = 100 cm 1 kg = 1000 g

1 cm = 10 mm 1 g = 1000 mg

Metric – Imperial

1 mile ≈ 1.6 km 1 kg ≈ 2.2 lbs

1 inch ≈ 2.5 cm 1 gallon ≈ 4.5 litres

1 yard ≈ 90 cm 1 litre ≈ 1.75 pints

UNITS SHOULD ALWAYS

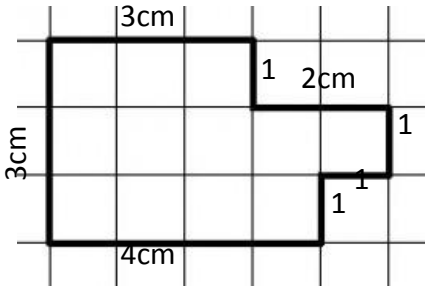
BE USED AS APPROPRIATE

Area and Perimeter

Perimeter – the distance around the outside edge of a shape (measured in cm, mm, m etc)

Area – the amount of space a 2D shape covers (measured in cm^2, mm^2, m^2 etc)

Example:

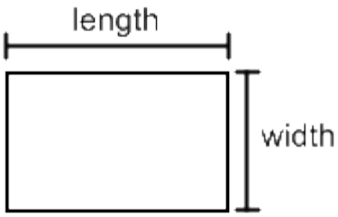


Perimeter = 3 + 1 + 2 + 1 + 1 + 1 + 4 + 3 = 16 cm

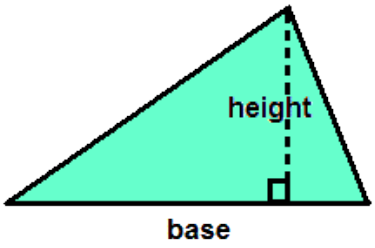
Area (by counting squares) = 12 cm^2

Areas of quadrilaterals and triangles:

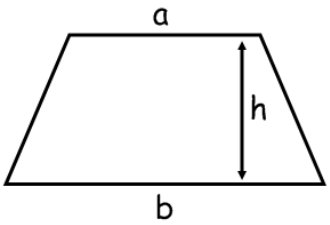
Area of rectangle = length x width



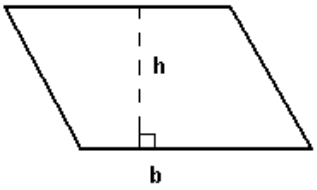
Area of triangle = ½ x base x perpendicular height



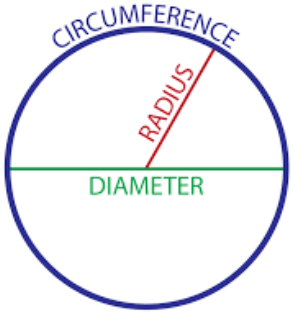
Area of trapezium = ½ x (a + b) x h



Area of parallelogram = base x perpendicular height



Circles



Circumference = π x diameter

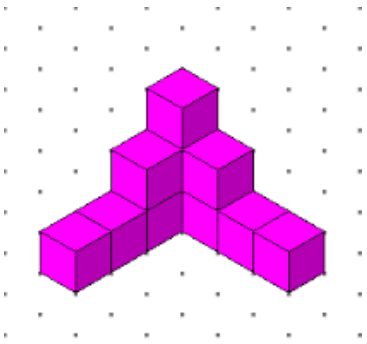
Area = π x r^2

Note: π is approximately 3.14, or can be used exactly using the π button on a calculator

Volume

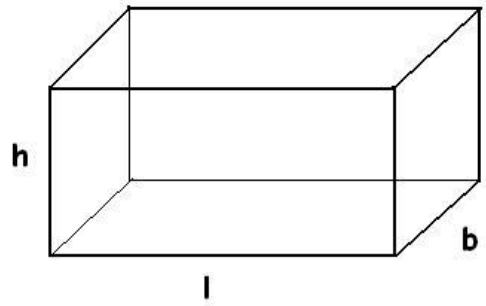
Volume is the amount of space a 3D shape occupies (measured in m^3, cm^3, mm^3)

Example:



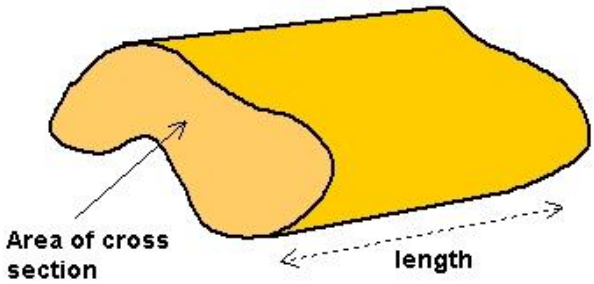
Volume (by counting cubes) = $11cm^3$

Cuboids



Volume = h x l x b

Prisms



Volume = area of cross-section x length

DATA HANDLING

Averages:

Mode - most common value in a set of data
Median – middle value when data set in order
Mean – the sum of all values divided by the number of values in the data set.

Range – describes the **spread** of the data (not the average of the data)

Range = highest value – lowest value

Statistical Diagrams:

Wherever possible the interpretation of graphs should be of utmost importance.

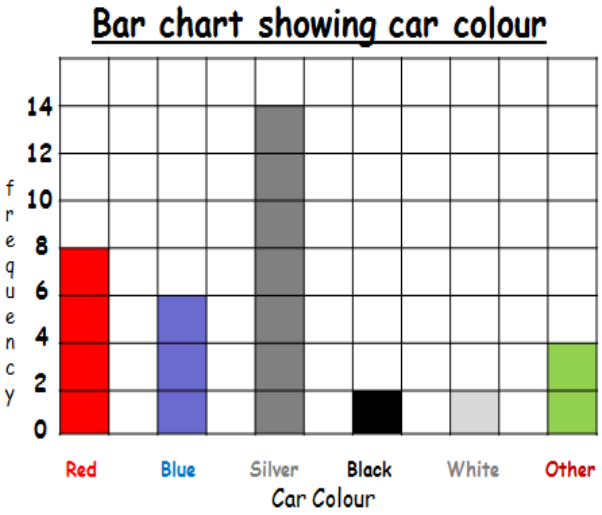
All diagrams should have the following:

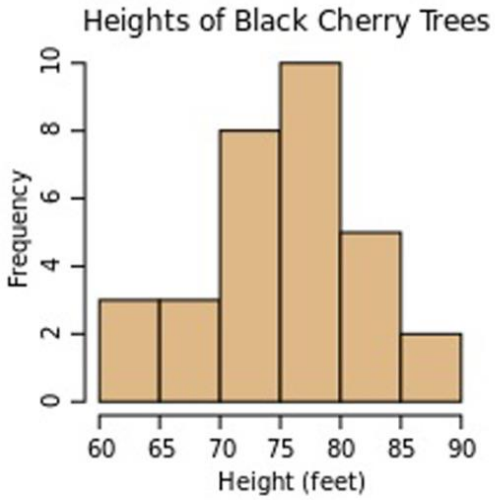
- Title
- Both axes labelled
- Graph breaks used to show where an scale doesn't start at zero
- Scales equally spaced
- Make good use of available space
- Where appropriate have independent variable on the horizontal axis, and dependent on the vertical

Bar Charts/Frequency diagrams

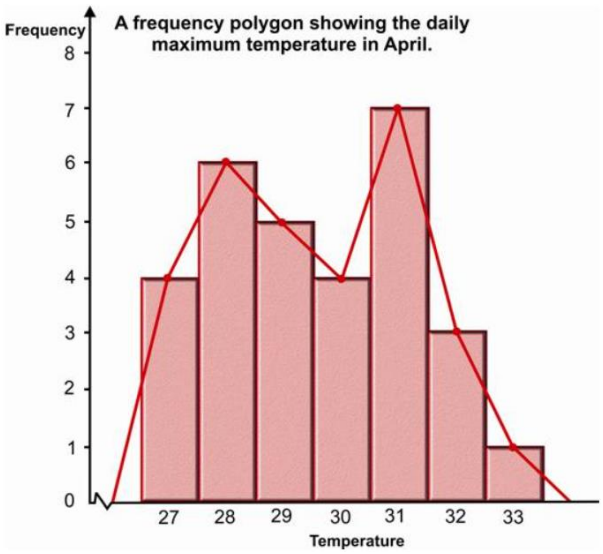
Bars must be of equal width
If data qualitative (words) or discrete (exact values) then leave spaces between bars. Label bars.
If data continuous no spaces. Label lines at edges of bars.

Examples:





Frequency Polygon: A frequency diagram where the middle of each bar is joined.

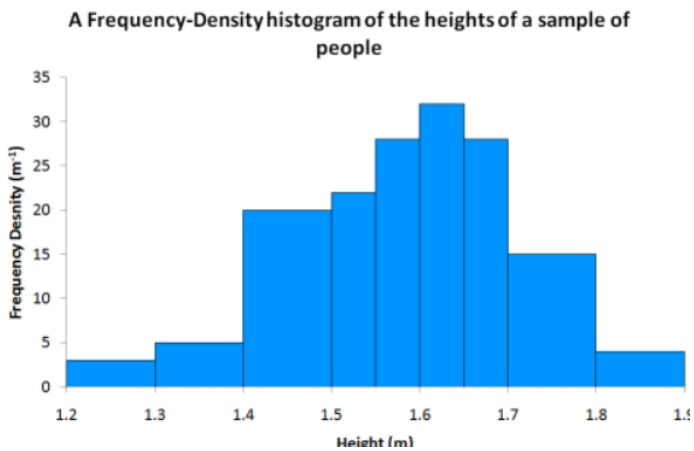


Histogram: A chart similar to a bar chart, but the area of the bar varies with the data rather than the height.
Generally the vertical axis is frequency density where

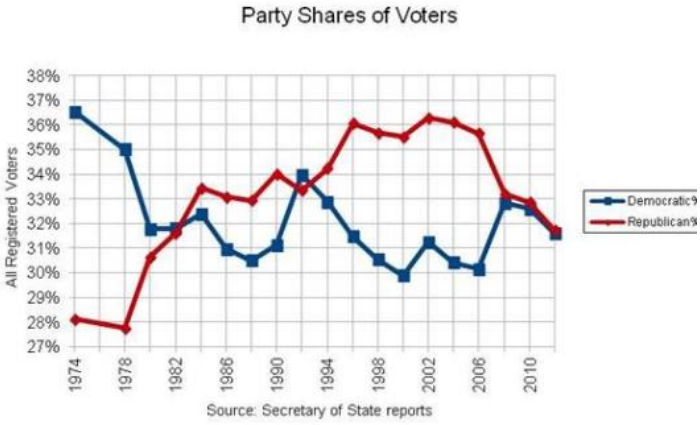
$$\text{Frequency density} = \frac{\text{Frequency}}{\text{Class width}}$$

Bars do NOT need to be the same width

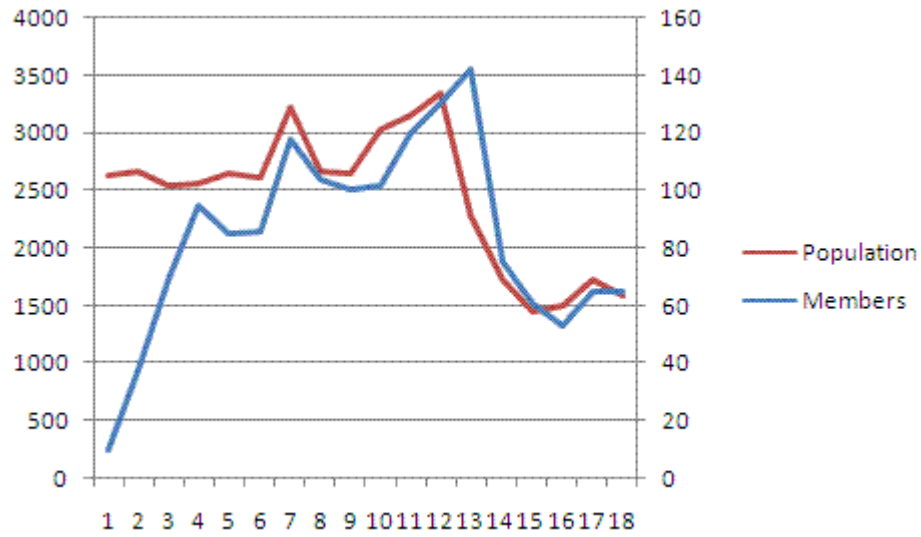
Area = frequency



Line graphs

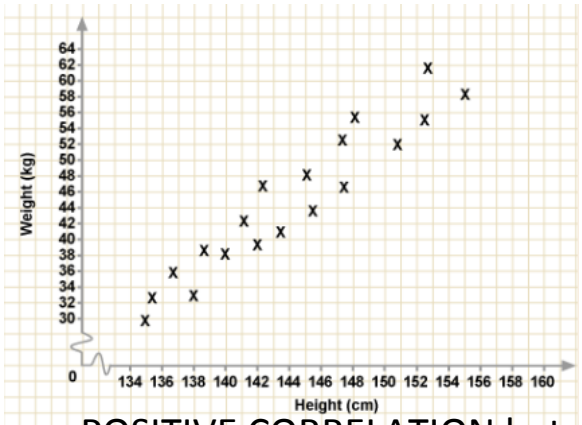


Dual Axis Graph: graph comparing two sets of data with different units of measure. Key essential



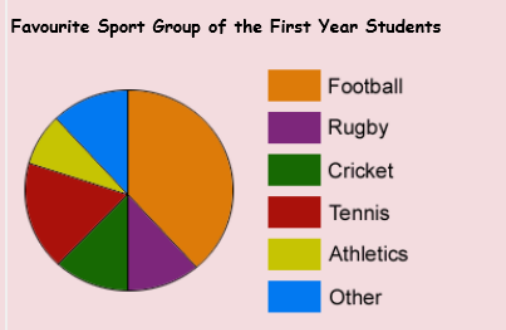
Scatter Graphs – used to see if there is a relationship between two variables:

Eg graph showing heights and weights of group of students



The graph shows a POSITIVE CORRELATION between height and weight.

Pie Charts: Used to see the proportions making up the whole



To work out appropriate angles for each sector:

$$\text{Angle} = \frac{\text{number in sector}}{\text{total frequency}} \times 360°$$

Stem-and-leaf: Shows distribution clearly.

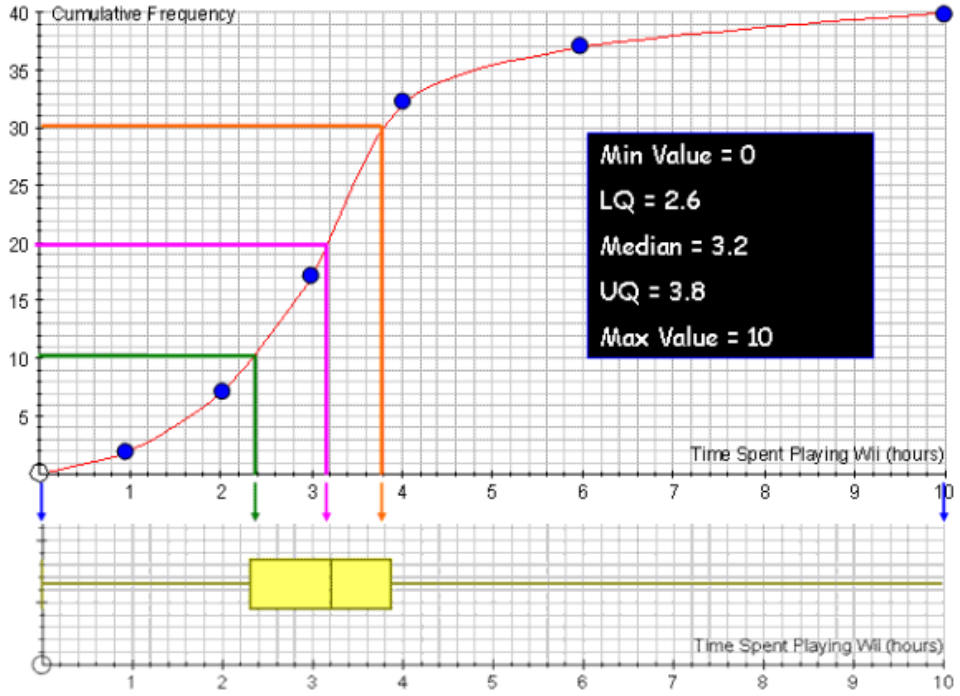
Stem and leaf showing ages at a party

stem	leaf
0	1, 1, 2, 2, 3, 4, 4, 4, 4, 5, 8
1	0, 0, 0, 1, 1, 3, 7, 9
2	5, 5, 7, 7, 8, 8, 9, 9
3	0, 1, 1, 1, 2, 2, 2, 4, 5
4	0, 4, 8, 9
5	2, 6, 7, 7, 8
6	3, 6

Key: 6|3 = 63 years old

Cumulative Frequency (running total) and Box Plots

Graphs showing number of time spent on the Wii



ALGEBRA

Basic rules:

a + a + a + a = 4a
2 x b = 2b
c x c = c^2 (not 2c)
d x d x 3 = $3d^2$
e x f x 4 = 4ef
g ÷ 5 is usually written as a fraction $\frac{g}{5}$

Collecting like terms – terms can be collected together if they are exactly the same

Examples:

3p + 2p = 5p
6a + 2b cannot be simplified
5a + 3b + 2a – 2b = 7a + b (note: b means 1b)

Brackets - To *expand brackets* (or *multiply out* brackets) make sure **everything** inside the bracket is multiplied by the value outside.

Eg: 4(a + 3) = 4xa + 4x3 = 4a + 12
a(2a – 4) = ax2a + ax-4 = $2a^2$- 4a

Factorising – this means writing an expression (or numbers) in terms of it's factors – often involves putting brackets back in.

Eg. 12a + 18 = 6(2a + 3)

Note: Always take the biggest factor outside the bracket.
(e.g 12a + 18 = 2(6a + 9) not fully factorised)

Solving equations/rearranging formulae

To solve an equation or to rearrange a formulae always use the BALANCING METHOD – ie at each step do the same to both sides.

Eg1 Solve

$$4a + 3 = 23$$
(-3) (-3)
$$4a = 20$$
(÷4) (÷4)
$$a = 5$$

Eg2 Make u the subject

$$v^2 = u^2 + 2as$$
(-2as) (-2as)
$$v^2 - 2as = u^2$$
(√) (√)
$$\sqrt{v^2 - 2as} = u$$

For further assistance:

MyMaths.co.uk is a useful website that covers all KS3 and KS4 topics very clearly and may be of assistance, alternatively please do not hesitate to contact King Edward VI Maths staff at any time to discuss any problems.